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MATHEMATICAL MODELLING OF WATER REGULATION PROCESSES ON DUAL-ACTION DRAINAGE SYSTEMS

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Abstract. *The solution of the problem of increasing water regulation areas in the Polissia zone of Ukraine requires investigation and development of new, more effective methods for determining structural parameters of drainage systems when developing projects for their reconstruction in accordance with the requirements aimed at ensuring water regulation during systems' operation. The paper considers the problem of improving the efficiency of water regulation on dual-action drainage systems by using mathematical modelling tools to determine the structural parameters of the systems and the parameters of their operational management. The proposed means are based on the use of Richards equation stated in terms of water head. As a tool for scenario modelling, an initial-boundary value problem of modelling moisture transfer on dual-action systems is formulated and a finite-difference scheme for obtaining its numerical solution is given. We consider the problem of determining the depth of drains installation and the distance between them at which the system provides not only the drainage of soil's surface layer, but also the maintenance of its moisture supply level in a given range with a minimum need for irrigation during the growing season. The algorithm for solving such a problem is presented. It is based on the construction of a set of admissible values of system's parameters using, in particular, the bisection method, followed by the minimization of an objective function on this set. Under the conditions when the implementation of underground water supply technology is economically impractical, the possibility of supplementing the drainage system with an irrigation system is considered. In this case, the cost of building a drainage system and an additional irrigation system is a criterion for the optimality of system's parameters. Additionally, we consider the problem of operational management of water regulation, i. e., the determination, given the initial distribution of moisture, of the optimal control influences necessary to ensure an acceptable level of moisture availability during a given period of time. This minimization problem is proposed to be solved by a genetic algorithm. The results of modelling the operation of a dual-action system and the optimization of its parameters under the conditions of drained peat soils of the Panfyly Research Station (Ukraine, Kyiv region) are presented.*
Keywords*: water regulation, dual-action systems, mathematical modelling*

Actuality of research. The problem of increasing the efficiency of water regulation is relevant for the territory of Ukrainian Polissia where 3.2 million hectares of drainage systems of various types are concentrated. These systems, built in 1970–1980s, were primarily intended to solve the problem of removing excess water in spring and creating a sufficient capacity in vadose zone for the accumulation of summer precipitation, the amount of which at that time was sufficient for water supply of most crops grown on drained lands. As a result of climate change, due to which the territory of Ukraine is characterized by one of the highest rates of growth in the average annual temperature in the world, conditions with an insufficient level of natural moisture supply are now forming in Ukrainian Polissia starting from July. Therefore, the approved "Irrigation and Drainage Strategy in Ukraine for the Period Until 2030" provides for the implementation of measures for the reconstruction and modernization of existing drainage systems by supplementing them with the function of water regulation during the entire growing season.

Implementation of this function is ensured through the possibility of using groundwater by regulating its level, or conducting irrigation, or a combination of these two options. The effectiveness of water regulation largely depends on the applied decision-making support tools.

Literature review. The main source of predictive data for physically-based decision-making support in water regulation is mathematical modelling of moisture transfer. On its base, forecasts of the dynamics of moisture availability to plants, necessary for operational water regulation, are obtained, and scenarios are simulated within the growing season to support medium-term planning of reclamation systems' operation. Initial data for modelling are the data on the hydro-physical properties of soils, preferably obtained experimentally, and the data of field measurements of current state indicators of the “soil-plant-atmosphere” system.

The most commonly used class of models is based on the Richards equation [1]. The methods of solving direct problems stated for such models vary from analytical [2] to fully numerical [3]. In particular, when modelling changing groundwater level using the moving simulation domain’s boundary, one of the effective numerical methods is the method of conformal mappings [4].

Decision-making support systems for water management mostly focus on one of its components (irrigation or drainage) and consist usually of only scenario modelling or operational management tools (see, e. g., [5]).

To solve the considered problems on dual-action systems, the following principles of using mathematical modelling tools in decision-making support systems in irrigation described in [6; 7] are applicable:

- the use of the Richards equation stated in terms of pressure/water heads to more accurately determine the availability of moisture to plants and manage it exclusively in the root-containing zones of the soil taking into account the structure of plants’ root systems;
- adaptation of the model to actual conditions by laboratory determination of soil’s hydro-physical characteristics and introduction of empirical parameters into the model [6]. The values of these parameters are further selected by solving inverse problems to obtain the best possible description of moisture dynamics within wetting-drying cycles;
- the use of swarm intelligence methods [8] to determine the parameters of design and operation of reclamation systems based on scenario modelling.

The aim of the research is the development of mathematical modelling tools that can be used for the determination of the design and operational parameters of dual-action systems that combine the functions of drainage and irrigation.

Materials and methods. We consider the problem of modelling the dynamics of water heads in a soil massif, on which water regulation is carried out by a double-acting system, under the following conditions ([9], Fig. 26):

– drains are installed without a slope and connect two canals;

– drains are considered to be constantly completely filled with water;

– the same water level is maintained in both canals.

If there is a large distance between the canals, filtration from them can be neglected and, considering the uniform distribution of moisture along the drains, modelling can be carried out in a two-dimensional approximation by considering a section parallel to the canals, in the middle between them (Fig. 1). At the same time, the simulation domain can be limited to the zone of influence of one drain. The lower boundary of the domain is the confining bed.

The Richards equation stated in terms of water heads, which takes into account the transition of soil from unsaturated to saturated state, is used in the simulation in the form described in [10]:

$$\left[C(h,z)+\frac{\theta(h,z)}{\theta_s(z)}S_s(z)\right]\frac{\partial h}{\partial t}= \\ =\frac{\partial}{\partial x}\left(k(h,z)\frac{\partial h}{\partial x}\right)+ \\ +\frac{\partial}{\partial z}\left(k(h,z)_{z}\left(\frac{\partial h}{\partial}-1\right)\right)- \\ -S(x,z,t), \\ 0\le x\le L, 0\le z\le L_z, t\ge 0 \quad , \qquad (1)$$

where $h(x, z, t)$ is the water head, m, $C(h,z)=\frac{\partial\theta}{\partial h}$ is the differential moisture capacity, $\theta(x, z, t)$ is the volumetric soil moisture,%, $\theta_s(z)$ is the saturated moisture content,%, $S_s(z)$ is the specific storage,

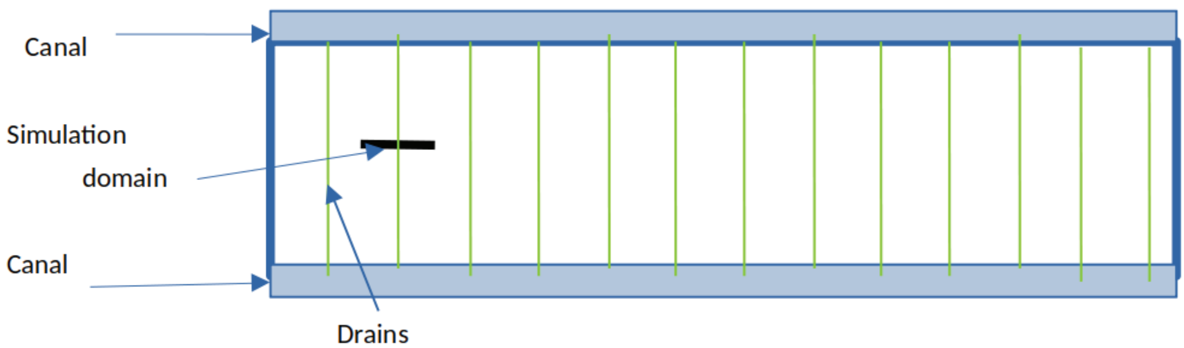

Fig. 1. Simulation domain

1/m, $k(h, z)$ is the hydraulic conductivity, *m/s*, *S(x, z, t)* is the function, *1/s*, that models moisture extraction by the roots of plants.

Soils are considered to have a layered structure. Their water retention curves are described according to the van Genuchten model [11] in the form $\theta(h)=\theta_0+\frac{\theta_1-\theta_0}{\left[1+\left(10\alpha|h|\right)^n\right]^{1-1/n}}$ with the values of the coefficients θ_r, θ_s, α, n that change from layer to layer. The dependency between hydraulic conductivity and water head is represented according to the Mualem model [12] in the form $k(h)=k_f\theta_r^{\beta}(h)\left[1-\left(1-\theta_r^{n/(n-1)}(h)\right)^{1-1/n}\right]^2$, $\theta_r(h)=\frac{\theta(h)-\theta_0}{\theta_1-\theta_0}$, where k_f is the saturated hydraulic conductivity, β is a fixed exponent.

The values of the coefficients of the models are determined by the least squares fitting of the data of laboratory studies.

The specific storage is also estimated from this data as $S_s=-\frac{\theta(h_1)-\theta(h_0)}{h_1-h_0}$ where $h_0 > h_1$ are the measured water heads that correspond to the close-to-saturated state of soil.

The width L of the simulation domain is equal to the distance between drains. One drain is modelled. It is considered to be located at a point with $x = L / 2$. At the location of the drain, water head equal to the water level in the canals above the drain is set [4; 13]. We assume that water head in the drain is fixed along its length.

The level *l, m,* of water in the canals changes taking into account the flows in the drains *Q, m/s*, and the evaporation from water surface *E, m/s*, as follows:

$$\frac{dl}{dt}=k_1Q-E,\ k_1=\frac{2\pi R\cdot l_d\cdot k_d}{w_cL},$$

where R is the radius of the drain, l_d is the length of the drain, w_c is the width of the canals (their shape is considered rectangular), k_d is the coefficient of linear dependency between pressure and flow in the drain. The evaporation from water surface is considered proportional to the evaporation from soil surface.

To Equation (1) we set the following boundary condition on soil surface $z = 0$ [14; 17]:

$$k(h,z)\frac{\partial h}{\partial z}=Q_e(t)-Q_p(t)-Q_i(x,t),$$

where $Q_e(t)$, $Q_p(t)$, $Q_i(x, t)$ are the fluxes, *m/s*, of evaporation, precipitation, and irrigation. The absence of flux condition is set on the side faces of the domain, and the condition $\frac{dh}{dx}=1$ is set on the bottom face.

The function S models the extraction of moisture by the root systems of plants the way it is described in [14]. Here the distribution of transpiration on the depth z is described according to [18] in the form $S_z(z,t)=\frac{T(t)L(z)}{\int_0^{z_r}L(z)dz}$, *m/s*, where z_r is the depth of the root-containing layer, $L(z)=1.44-0.14\frac{z}{z_r}-0.61\left(\frac{z}{z_r}\right)^2-0.69\left(\frac{z}{z_r}\right)$ is the function of the distribution of root length density and its specific form used in this paper, $T(t)$ is the transpiration rate, *m/s*. We assume that n_p plants with the centres of root systems, the depth of which is equal to r_{pi}, located in the points x_{pi}, $i = 0$ ..., $n_p - 1$in the simulation domain. The density of root systems is assumed to decrease linearly subject to the horizontal coordinate x that is described by the function

$$S_{xi}(x)=\begin{cases}\frac{r_{pi}-(x-x_{pi})}{r_{pi}^2}, & r_{pi}-(x-x_{pi})\ge 0\\ 0, & r_{pi}-(x-x_{pi})<0\end{cases},\quad 1/m.$$

Then the total moisture extraction function has the form $S(x,z,t)=\frac{1}{n_p}S_z(z,t)\sum_{i=0}^{n_p-1}S_{xi}(x)$, *1/s*.

Numerical solution of the initial-boundary value problem for the model based on Equation (1) is performed according to the implicit finite-difference Crank-Nicholson scheme [19] on a uniform grid with respect to the space variables with the use of the algorithm for adaptive time step selection [15].

Similarly to [15], we consider the uniform finite-difference grid

$$\omega=\left\{(x_i=ih_x, z_k=kh_z, t_j=j\tau): : i=\overline{0,m},\ k=\overline{0,n},\ j=0,1,2,...\right\},$$

where h_x, h_z are the steps with respect to the spatial variables, τ is the time step. Here and further the grid analogue of the water head function h and, similarly, other functions, is designated as $h_{ik}^j=h(x_i,z_k,t_j)$. As the results of discretization we obtain [15] the following linear system that is further solved by the TFQMR algorithm [20]:

$$h_{i-1,k}^j\times A_{1,i,k}^{j-1}+h_{i,k-1}^j\times A_{2,i,k}^{j-1}+h_{i+1,k}^j\times \times B_{1,i,k}^{j-1}+h_{i,k+1}^j\times B_{2,i,k}^{j-1}-h_{i,k}^j\times R_{i,k}^{j-1}=\Phi_{i,k}^{j-1},$$

$$A_{1,i,k}^{j-1} = \frac{1}{4h_x^2}\left(k\left(h_{i-1,k}^{j-1}\right) + k\left(h_{i,k}^{j-1}\right)\right),$$

$$A_{2,i,k}^{j-1} = \frac{1}{4h_z^2}\left(k\left(h_{i,k-1}^{j-1}\right) + k\left(h_{i,k}^{j-1}\right)\right),$$

$$B_{1,i,k}^{j-1} = \frac{1}{4h_x^2}\left(k\left(h_{i+1,k}^{j-1}\right) + k\left(h_{i,k}^{j-1}\right)\right),$$

$$B_{2,i,k}^{j-1} = \frac{1}{4h_z^2}\left(k\left(h_{i,k+1}^{j-1}\right) + k\left(h_{i,k}^{j-1}\right)\right),$$

$$R_{i,k}^{j-1} = A_{1,i,k}^{j-1} + A_{2,i,k}^{j-1} + B_{1,i,k}^{j-1} + B_{2,i,k}^{j-1} + \\ + \frac{1}{\tau}\left(C\left(h_{i,k}^{j-i}\right) + \frac{\theta\left(h_{i,k}^{j-1}, z_k\right)}{\theta_s\left(z_k\right)} S_s\left(z_k\right) \right),$$

$$\Phi_{i,k}^{j-1} = -h_{i-1,k}^{j-1} \times A_{1,i,k}^{j-1} - h_{i,k-1}^{j-1} \times A_{2,i,k}^{j-1} - \\ - h_{i+1,k}^{j-1} \times B_{1,i,k}^{j-1} - h_{i,k+1}^{j-1} \times B_{2,i,k}^{j-1} + \\ + \left(\begin{array}{l} A_{1,i,k}^{j-1} + A_{2,i,k}^{j-1} + B_{1,i,k}^{j-1} + B_{2,i,k}^{j-1} - \\ - \frac{1}{\tau}\left(C\left(h_{i,k}^{j-1}\right) + \frac{\theta\left(h_{i,k}^{j-1}, z_k\right)}{\theta_s\left(z_k\right)} S_s\left(z_k\right) \right) \end{array} \right) \times \\ \times h_{i,k}^{j-1} - S_{i,k}^{j} + \frac{1}{2h_z}\left(k\left(h_{i,k+1}^{j-1}\right) - k\left(h_{i,k-1}^{j-1}\right)\right)$$

$$h_{0k} = h_{1k}, h_{mk} = h_{m-1k}, k = \overline{0,n},$$

$$H_{in} = H_{in-1} + h_z, i = \overline{0,m},$$

$$H_{i0} = H_{i1} - \frac{h_z\left(Q_e + Q_p + Q_i\right)}{\frac{1}{2}\left(k_{i0} + k_{i1}\right)}.$$

To adapt the model to the actual conditions of the processes, the coefficients of the van Genuchten and Mualem models can be varied to achieve a better correspondence of the simulation results to the measured water head dynamics. In particular, the procedure of applying the particle swarm optimization approach for solving such a problem is given in [6].

In the process of modelling, we calculate the minimum and maximum water head levels in the root zone, averaged in each column of grid nodes with the values of root system's distribution function as weights.

Modelling of water regulation in order to increase water content in the root zone of the soil is modelled as follows. At first the increase in water level in the canals is simulated for no more than a given period of time. After that, the simulation is carried out without control actions until the maximum water head level stops increasing and the minimal one starts increasing (thus determining the maximum influence of the increase in water level in the canals on moisture content in the root layer).

If the maximum water head level is lower than the upper limit of the maintained range or the minimum water head level is lower than the pre-irrigation threshold, then the surface supply of irrigation water is simulated, similarly, for no more than a given period of time.

When the maximum water head level in the root zone increases above the threshold value for which drainage is necessary, a decrease in water level in the canals is simulated.

The controlled change in water level in the canals is simulated with a given fixed speed.

Results. The main problem that should be solved by decision support systems when designing dual-action drainage systems consists of determining their main constructive parameters, in particular, the depth of installation and the distance between drains.

Given the known characteristics of drainage pipelines, we assume that a double-action system fulfils its functions if, by setting a fixed water level in the canals, it allows

- for a given period of time, transfer the upper layer of the soil of a given thickness from saturated to non-saturated state in order to ensure at the beginning of the season the passability of machinery that must carry out soil cultivation;
- at a fixed low level of evapotranspiration at the initial stages of plant development, ensure a given range of moisture availability in the root layer without additional irrigation;
- at a fixed high level of evapotranspiration and the maximum level of root system's development, similarly to the initial stages, ensure the availability of moisture.

In the case when the system is not able to provide water consumption for plants, the possibility of additional construction of a surface irrigation system should be considered.

The objective function to be minimized is the cost of constructing a dual-action system, which consists of the cost of drainage pipelines, the cost of their installation, and, if necessary, the cost of constructing a surface irrigation system.

The proposed decision-support algorithm consists in finding a set of values of system's constructive parameters, under which it properly performs at least the drainage function, followed by finding, by the greedy algorithm, of such values from this set that minimize the objective function.

Building of a set of allowable values of constructive parameters under known hydro-physical properties of the soil is performed according to the following algorithm:

• the depth of drains installation decreases with a given step starting from the depth of the bottom of the canals and ending with the depth of the layer that needs drainage. For each depth value three tests are performed:

◦ test 1: the maximum distance between drains at which the system performs the drainage function at the beginning of the season is determined (initial water head distribution $h= -z$ corresponds to fully saturated soil, the water head in the drain is set to be constant at the level of $h=0$). We assumed that the dependency between the inter-drain distance and the depth of the drained layer is monotonic and solve the corresponding problem by the bisection method. If drainage cannot be carried out at the minimum inter-drain distance, we proceed to the next depth of drains installation;

◦ test 2: for the distance between drains determined on the previous step, the minimum value of the fixed maintained water level in the canals is found by the bisection method, for which, in a close-to-steady state, the minimum weighted average water head in the vertical section of the root zone is greater than the level of pre-irrigation threshold in a situation of low water consumption. The test is considered passed if the above-described condition regarding the maximum weighted average water head in the root zone is met;

◦ test 3: similar modelling is carried out for the situation of high water consumption;

◦ in the case when tests 2 or 3 are not passed, the cost of constructing a surface irrigation system is added to the value of the objective function.

For an active dual-action system, which contains both a network of drainage pipelines and a surface irrigation system, the problem of operational management is also considered. It consists in finding, for a given initial distribution of water heads, the control actions (changes in water level in the canals and, if necessary, surface irrigation) necessary to maintain moisture availability in a given range during a given period of time. Given the known cost of water level regulation and the cost of surface irrigation application we determine a regime (using the genetic algorithms approach [16]), in which the total cost of water regulation is minimal.

Since water regulation by changing water level in the canals has a delayed effect on the moisture content of the root layer of the soil, on the first step of the algorithm we determine the time when, in the absence of water regulation, the minimum weighted average water head level in the vertical section will become less than the pre-irrigation threshold. Water regulation is further modelled in a way to finish at this specific moment of time. If, after water regulation is performed, water heads do not belong to the maintained range, a given large penalty value is added to the value of the objective function.

The operation of a dual-action system was simulated for the conditions of drained peat soil of the Panfyly experimental station (Ukraine, Kyiv region, 50°13'16.9"N31°45'46.4"E).

The coefficients of the van Genuchten and Mualem models for soil layers, obtained on the base of experimental study, are presented in Table 1.

The model parameters had the following values:

• The radius of the drain is 0.2 m, the length of the drain (distance between canals) is 150 m. The depth of simulation domain is equal to 3 m.

• The depth of the canal is 2 m. The width of the canal is 3 m.

• The maintained range of average water heads in the root zone is –40 – –15 kPa. Drainage by lowering water level in the canals is carried out when the average water head level in the root zone is greater than –5 kPa.

• Evapotranspiration for the high water consumption case is equal to 5 mm/day and for the low level case – to 2 mm/day. The distance between plants is 66 cm, the depth of the root system is 50 cm at the high water consumption case, and 20 cm at the low water consumption case.

• The rate of change in water level in the canals is 2 m/day. The flow of irrigation water is 7 mm/hour.

• The cell size of the finite-difference grid is 10×10 cm, the maximum length of the time step is 100 s.

The results obtained by applying the procedure for determining the permissible values of system’s constructive parameters, provided that at the beginning of the season the minimum depth of the saturated zone should become more than 40 cm due to the drainage during no more than 5 days, are shown in Fig. 2 and 3.

1. Coefficients of the van Genuchten and Mualem models

Layer	θ_r	θ_s	α	n	k_f, m/s	β	S_s, 1/m
0.05–0.2 m	0.37	0.81	0.1	1.5	2.3×10^{-4}	-0.75	0.0823
0.2–0.4 m	-0.5	0.87	0.01	1.1	6.0×10^{-6}	-15	0.0186
0.4–0.6 m	-1.5	0.962	0.01	1.1	6.0×10^{-6}	-20	0.0063
0.6–0.8 m	-2	0.957	0.01	1.1	1.0×10^{-5}	-15	0.0156

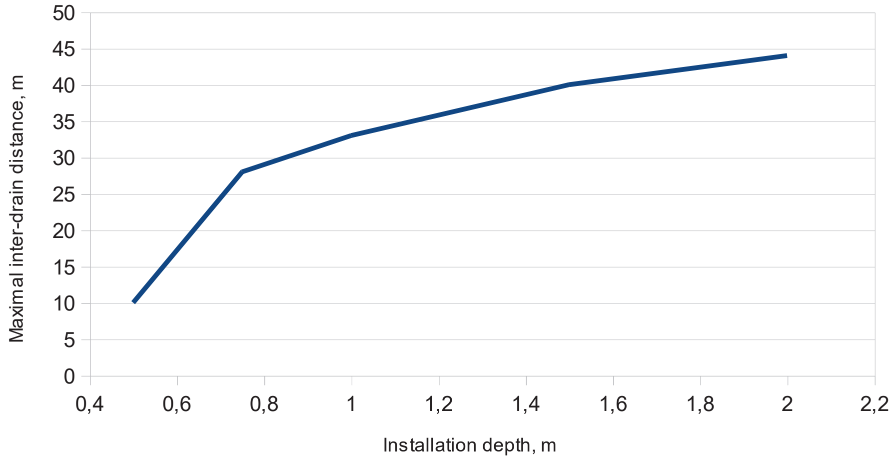

Fig. 2 The maximum inter-drain distance subject to the depth of installation

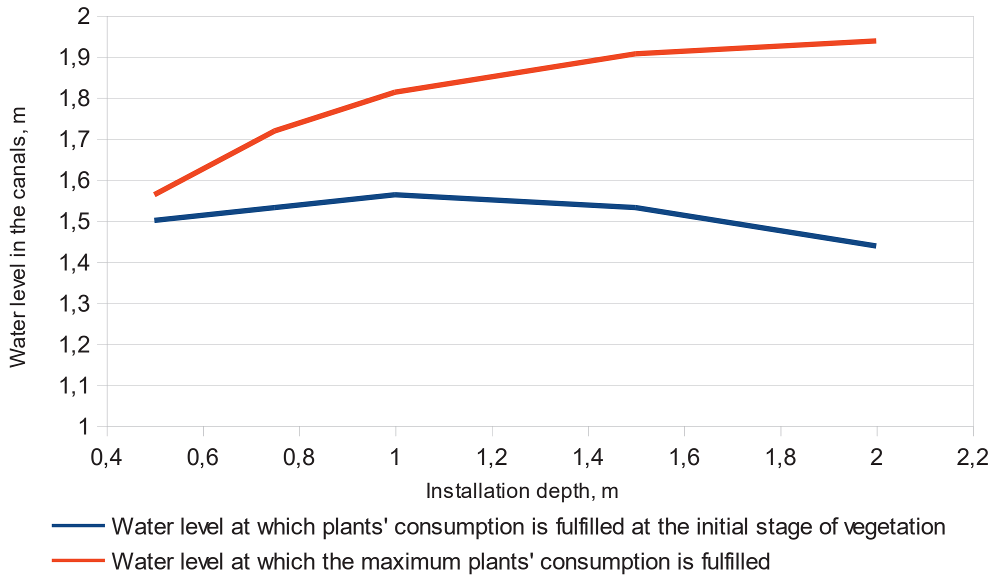

Fig. 3. Water levels in the canals at which plant's water consumption is fulfilled at the specified maximum distance between drains

For the installation depth of more than 75 cm, with high water consumption level, ensuring the required level of moisture availability in zone between drains was accompanied by the saturation of soil by water in the vertical section at the location of the drain. Thus, the depth of 75 cm with the inter-drain distance of 28 m was determined as the greatest depth of drains installation, at which the dual-action system effectively performs both the functions of drainage and irrigation.

This set of admissible values of system's constructive parameters was used to simulate operational assignment of control actions to maintain the moisture availability in the root layer in a given range. With constant evapotranspiration at the level of 3 mm/day, which is greater than the level at which the system performs the function of irrigation without the use of surface irrigation, and the depth of the root system equal to 20 cm, the water level in the canals was modelled as being maintained at the level of at least 1.53 m. Initial distribution of water heads was taken as $h = -0.4 - z$.

In the absence of control actions, the need for water regulation according to the simulation results arises in 3.3 days after the starting moment of time. The determined optimal regime of maintaining the given water head range consisted in increasing the water level in the canals at the time of 1.74 days for 1.43 days. In such case, water regulation is possible without additional use of surface irrigation.

The dynamics of weighted averaged water heads in vertical section is shown in Fig. 4. In the considered situation, water heads in the vertical

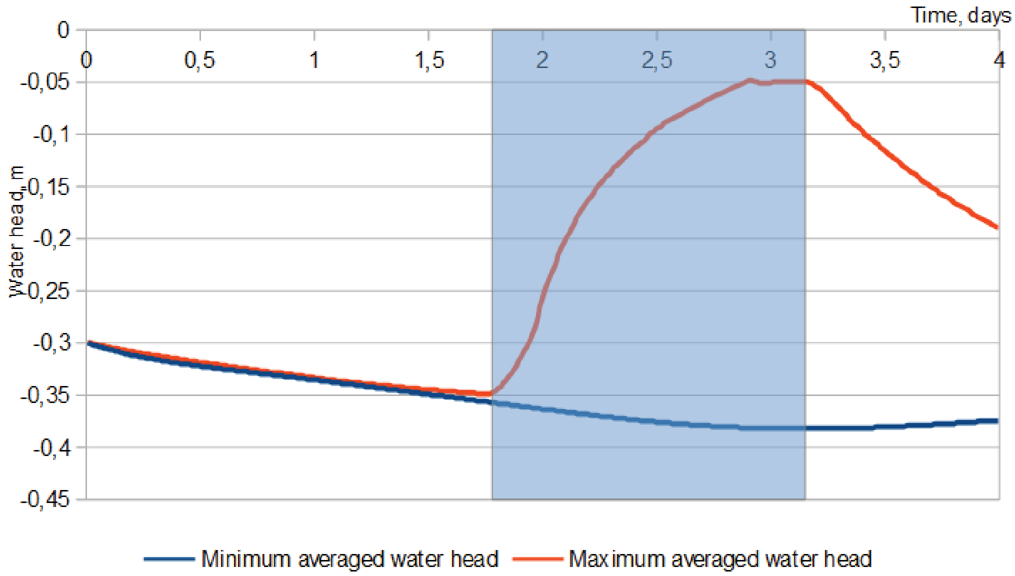

Fig. 4 Dynamics of averaged water heads in vertical section
(water regulation period is marked with colour)

section at the point of drain installation rise in the process of water regulation to the level at which the drainage procedure is activated in the model (–5 kPa) while moisture content in the zone between drains still continues to decrease. As a result, the two applied control actions do the opposite – a decrease and an increase of water level in the canals – that lead to the fixation of this level together with the maximum average water head in vertical section in the period of time preceding the completion of water regulation.

Conclusions. We propose to determine the technologically effective and economically feasible values of such constructive parameters of double-action drainage systems as the installation depth and the distance between drains using mathematical modelling methods on the base of the Richards equation stated in terms of water head.

The calculated values of parameters are considered admissible if the system is able to drain water within a specified period to the depth sufficient for the operation of agricultural machinery in spring, and allows maintaining, through vertical flow of moisture, the moisture supply of the root layer of the soil during the period of maximum water consumption by plants not below the lower limit of the range of optimal moisture supply without the use of irrigation.

Under the conditions when the implementation of such a technology is economically impractical, the possibility of supplementing the drainage system with an irrigation system is considered. In this case, the criterion for the optimality of system's parameters is the cost of building a drainage system and an additional irrigation system. In order to determine the operational necessity of applying irrigation, a suitable decision support algorithm is proposed.

Approbation of the methodology on the data of the existing drainage system demonstrates the adequacy of the obtained modelling results.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСІВ ВОДОРЕГУЛЮВАННЯ НА ДРЕНАЖНИХ СИСТЕМАХ ПОДВІЙНОЇ ДІЇ

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Анотація. *Вирішення завдання нарощування площ водорегулювання в зоні українського Полісся потребує пошуку та розроблення нових, більш ефективних методів визначення конструктивних параметрів дренажних систем при розробленні проєктів їх реконструкції згідно з вимогами забезпечення водорегулювання в процесі експлуатації. Розглядається задача підвищення ефективності водорегулювання на дренажних системах подвійної дії шляхом застосування засобів математичного моделювання для визначення конструктивних параметрів систем та оперативного управління ними. Пропоновані засоби базуються на використанні рівняння Річардса, записаного у термінах напорів. Як засіб сценарного моделювання, сформульовано початково-крайову задачу моделювання вологоперенесення на системах подвійної дії та наведено скінченно-різницеву схему отримання її чисельного розв'язку. Розглядається задача визначення таких глибини закладання дрен та відстані між ними за яких система забезпечує не тільки дренування поверхневого шару ґрунту, але й підтримання рівня його вологозабезпечення в заданому діапазоні за мінімальної необхідності застосування зрошення протягом сезону вегетації. Наведено алгоритм розв'язання такої задачі, який базується на побудові множини допустимих значень параметрів системи використовуючи, зокрема метод бісекції, з подальшою мінімізацією цільової функції на цій множині. За умов, коли реалізація технології підґрунтового живлення є економічно недоцільною, розглядається можливість доповнення системи дренажу системою зрошення. Критерієм оптимальності параметрів системи в цьому випадку є затрати на будівництво системи дренажу і додаткової системи зрошення. Також розглядається задача оперативного управління водорегулюванням – визначення за заданого початкового розподілу вологи оптимальних керуючих впливів, необхідних для забезпечення допустимого рівня доступності вологи протягом заданого періоду часу. Цю задачу мінімізації пропонується розв'язувати генетичним алгоритмом. Наведено результати моделювання роботи системи подвійної дії та оптимізації її параметрів за умов осушуваних торфових ґрунтів Панфильської дослідної станції (Україна, Київська область).*
Ключові слова: *водорегулювання, системи подвійної дії, математичне моделювання*